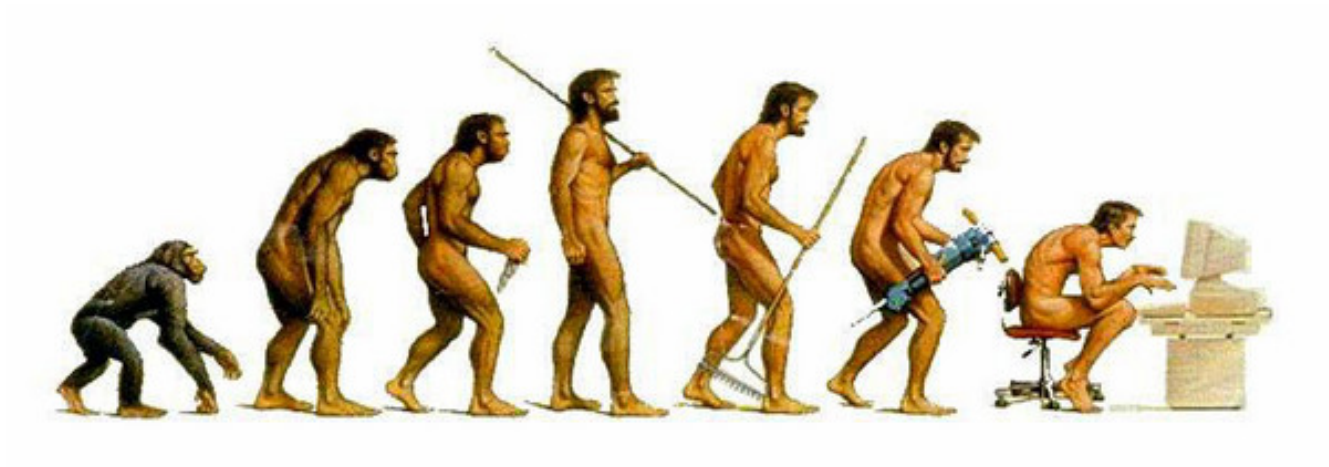


Faith, Evolution, and Programming Languages

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TechMesh, London
4 December 2012



Evolution



Multiculturalism

Part I

Church: The origins of faith

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Gerhard Gentzen (1909–1945)



Gerhard Gentzen (1935) — Natural Deduction

$$\begin{array}{c} \supset -I \\ \begin{array}{c} [A] \\ B \end{array} \\ \hline A \supset B \end{array} \qquad \begin{array}{c} \supset -E \\ A \quad A \supset B \\ \hline B \end{array}$$

$$\begin{array}{c} \& -I \\ A \quad B \\ \hline A \& B \end{array} \qquad \begin{array}{c} \& -E \\ A \& B \quad A \& B \\ \hline A \quad B \end{array}$$

Gerhard Gentzen (1935) — Natural Deduction

$\&-I$ $\frac{\mathcal{A} \quad \mathcal{B}}{\mathcal{A} \& \mathcal{B}}$	$\&-E$ $\frac{\mathcal{A} \& \mathcal{B}}{\mathcal{A}} \quad \frac{\mathcal{A} \& \mathcal{B}}{\mathcal{B}}$	$\vee-I$ $\frac{\mathcal{A}}{\mathcal{A} \vee \mathcal{B}} \quad \frac{\mathcal{B}}{\mathcal{A} \vee \mathcal{B}}$	$\vee-E$ $\frac{\mathcal{A} \vee \mathcal{B} \quad \begin{array}{c} [\mathcal{A}] \\ \mathcal{C} \end{array} \quad \begin{array}{c} [\mathcal{B}] \\ \mathcal{C} \end{array}}{\mathcal{C}}$
$\forall-I$ $\frac{\mathfrak{F}a}{\forall x \mathfrak{F}x}$	$\forall-E$ $\frac{\forall x \mathfrak{F}x}{\mathfrak{F}a}$	$\exists-I$ $\frac{\mathfrak{F}a}{\exists x \mathfrak{F}x}$	$\exists-E$ $\frac{\exists x \mathfrak{F}x \quad \begin{array}{c} [\mathfrak{F}a] \\ \mathcal{C} \end{array}}{\mathcal{C}}$
$\supset-I$ $\frac{\begin{array}{c} [\mathcal{A}] \\ \mathcal{B} \end{array}}{\mathcal{A} \supset \mathcal{B}}$	$\supset-E$ $\frac{\mathcal{A} \quad \mathcal{A} \supset \mathcal{B}}{\mathcal{B}}$	$\neg-I$ $\frac{\begin{array}{c} [\mathcal{A}] \\ \wedge \end{array}}{\neg \mathcal{A}}$	$\neg-E$ $\frac{\mathcal{A} \quad \neg \mathcal{A}}{\wedge} \quad \frac{\wedge}{\mathfrak{D}} .$

Gerhard Gentzen (1935) — Natural Deduction

$$\frac{\begin{array}{c} [A]^x \\ \vdots \\ B \end{array}}{A \supset B} \supset\text{-I}^x \qquad \frac{A \supset B \quad A}{B} \supset\text{-E}$$

$$\frac{A \quad B}{A \& B} \&\text{-I} \qquad \frac{A \& B}{A} \&\text{-E}_0 \qquad \frac{A \& B}{B} \&\text{-E}_1$$

Simplifying a proof

$$\frac{\frac{\frac{[B \& A]^z}{A} \&-E_1 \quad \frac{[B \& A]^z}{B} \&-E_0}{A \& B} \&-I \quad \frac{[B]^y \quad [A]^x}{B \& A} \&-I}{(B \& A) \supset (A \& B) \supset -I^z \quad B \& A} \supset -E$$
$$A \& B$$

Simplifying a proof

$$\begin{array}{c}
 \frac{[B \& A]^z}{A} \&-E_1 \quad \frac{[B \& A]^z}{B} \&-E_0 \\
 \hline
 A \& B \quad \&-I \\
 \hline
 (B \& A) \supset (A \& B) \supset-I^z \\
 \hline
 \frac{(B \& A) \supset (A \& B) \quad \frac{[B]^y \quad [A]^x}{B \& A} \&-I}{A \& B} \supset-E
 \end{array}$$

$\Downarrow$

$$\begin{array}{c}
 \frac{[B]^y \quad [A]^x}{B \& A} \&-I \quad \frac{[B]^y \quad [A]^x}{B \& A} \&-I \\
 \hline
 \frac{B \& A}{A} \&-E_1 \quad \frac{B \& A}{B} \&-E_0 \\
 \hline
 A \& B \quad \&-I
 \end{array}$$

Simplifying a proof

$$\begin{array}{c}
 \frac{[B \& A]^z}{A} \&-E_1 \quad \frac{[B \& A]^z}{B} \&-E_0 \\
 \hline
 A \& B \quad \&-I \\
 \hline
 (B \& A) \supset (A \& B) \supset-I^z \\
 \hline
 \frac{(B \& A) \supset (A \& B) \quad \frac{[B]^y \quad [A]^x}{B \& A} \&-I}{A \& B} \supset-E
 \end{array}$$

$\Downarrow$

$$\begin{array}{c}
 \frac{[B]^y \quad [A]^x}{B \& A} \&-I \quad \frac{[B]^y \quad [A]^x}{B \& A} \&-I \\
 \hline
 \frac{B \& A}{A} \&-E_1 \quad \frac{B \& A}{B} \&-E_0 \\
 \hline
 A \& B \quad \&-I
 \end{array}$$

$\Downarrow$

$$\frac{[A]^x \quad [B]^y}{A \& B} \&-I$$

Alonzo Church (1903–1995)



Alonzo Church (1932) — Lambda calculus

An occurrence of a variable $\mathbf{x}$ in a given formula is called an occurrence of $\mathbf{x}$ as a *bound variable* in the given formula if it is an occurrence of $\mathbf{x}$ in a part of the formula of the form $\lambda \mathbf{x}[\mathbf{M}]$; that is, if there is a formula $\mathbf{M}$ such that $\lambda \mathbf{x}[\mathbf{M}]$ occurs in the given formula and the occurrence of $\mathbf{x}$ in question is an occurrence in $\lambda \mathbf{x}[\mathbf{M}]$. All other occurrences of a variable in a formula are called occurrences as a *free variable*.

A formula is said to be *well-formed* if it is a variable, or if it is one

Alonzo Church (1940) — Typed λ -calculus

$$\frac{\begin{array}{c} [x : A]^x \\ \vdots \\ u : B \end{array}}{\lambda x. u : A \supset B} \supset\text{-I}^x \qquad \frac{s : A \supset B \quad t : A}{st : B} \supset\text{-E}$$

$$\frac{t : A \quad u : B}{\langle t, u \rangle : A \& B} \&\text{-I}$$

$$\frac{s : A \& B}{s_0 : A} \&\text{-E}_0$$

$$\frac{s : A \& B}{s_1 : B} \&\text{-E}_1$$

Simplifying a program

$$\begin{array}{c}
 \frac{[z : B \ \& \ A]^z}{z_1 : A} \ \&-E_1 \quad \frac{[z : B \ \& \ A]^z}{z_0 : B} \ \&-E_0 \\
 \hline
 \frac{\langle z_1, z_0 \rangle : A \ \& \ B}{\lambda z. \langle z_1, z_0 \rangle : (B \ \& \ A) \supset (A \ \& \ B)} \ \&-I \\
 \hline
 \frac{\lambda z. \langle z_1, z_0 \rangle : (B \ \& \ A) \supset (A \ \& \ B) \quad \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \ \& \ A} \ \&-I}{(\lambda z. \langle z_1, z_0 \rangle) \langle y, x \rangle : A \ \& \ B} \supset-E
 \end{array}$$

Simplifying a program

$$\begin{array}{c}
 \frac{[z : B \& A]^z}{z_1 : A} \&-E_1 \quad \frac{[z : B \& A]^z}{z_0 : B} \&-E_0 \\
 \frac{\quad}{\langle z_1, z_0 \rangle : A \& B} \&-I \\
 \frac{\quad}{\lambda z. \langle z_1, z_0 \rangle : (B \& A) \supset (A \& B)} \supset-I^z \quad \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I \\
 \hline
 \frac{\quad}{(\lambda z. \langle z_1, z_0 \rangle) \langle y, x \rangle : A \& B} \supset-E
 \end{array}$$

$\Downarrow$

$$\begin{array}{c}
 \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I \quad \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I \\
 \frac{\quad}{\langle y, x \rangle_1 : A} \&-E_1 \quad \frac{\quad}{\langle y, x \rangle_0 : B} \&-E_0 \\
 \hline
 \frac{\quad}{\langle \langle y, x \rangle_1, \langle y, x \rangle_0 \rangle : A \& B} \&-I
 \end{array}$$

Simplifying a program

$$\begin{array}{c}
 \frac{[z : B \& A]^z}{z_1 : A} \&-E_1 \quad \frac{[z : B \& A]^z}{z_0 : B} \&-E_0 \\
 \hline
 \frac{\langle z_1, z_0 \rangle : A \& B}{\lambda z. \langle z_1, z_0 \rangle : (B \& A) \supset (A \& B)} \&-I \\
 \hline
 \frac{\lambda z. \langle z_1, z_0 \rangle : (B \& A) \supset (A \& B) \quad \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I}{(\lambda z. \langle z_1, z_0 \rangle) \langle y, x \rangle : A \& B} \supset-E
 \end{array}$$

$\Downarrow$

$$\begin{array}{c}
 \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I \quad \frac{[y : B]^y \quad [x : A]^x}{\langle y, x \rangle : B \& A} \&-I \\
 \hline
 \frac{\langle y, x \rangle : B \& A}{\langle y, x \rangle_1 : A} \&-E_1 \quad \frac{\langle y, x \rangle : B \& A}{\langle y, x \rangle_0 : B} \&-E_0 \\
 \hline
 \frac{\langle y, x \rangle_1 : A \quad \langle y, x \rangle_0 : B}{\langle \langle y, x \rangle_1, \langle y, x \rangle_0 \rangle : A \& B} \&-I
 \end{array}$$

$\Downarrow$

$$\frac{[x : A]^x \quad [y : B]^y}{\langle x, y \rangle : A \& B} \&-I$$

William Howard (1980) — Curry-Howard Isomorphism

THE FORMULAE-AS-TYPES NOTION OF CONSTRUCTION

W. A. Howard

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Illinois at Chicago Circle, Chicago, Illinois 60680, U.S.A.*

Dedicated to H. B. Curry on the occasion of his 80th birthday.

The following consists of notes which were privately circulated in 1969. Since they have been referred to a few times in the literature, it seems worth while to publish them. They have been rearranged for easier reading, and some inessential corrections have been made.



Curry-Howard



Hindley-Milner



Girard-Reynolds



Part II

Second-order logic, Polymorphism, and Java

Gottlob Frege (1879) — Quantifiers ($\forall$)

It is clear also that from

$$\vdash \frac{\Phi(a)}{A}$$

we can derive

$$\vdash \frac{\frac{a}{\Phi(a)}}{A}$$

*if A is an expression in which a does not occur and if a stands only in the argument places of $\Phi(a)$.*¹⁴ If $\frac{a}{\Phi(a)}$ is denied, we must be able to specify a meaning for a such that $\Phi(a)$ will be denied. If, therefore, $\frac{a}{\Phi(a)}$ were to be denied and

John Reynolds (1974) — Polymorphism

TOWARDS A THEORY OF TYPE STRUCTURE[†]

John C. Reynolds

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Introduction

The type structure of programming languages has been the subject of an active development characterized by continued controversy over basic principles.⁽¹⁻⁷⁾ In this paper, we formalize a view of these principles somewhat similar to that of J. H. Morris.⁽⁵⁾ We introduce an extension of the typed lambda calculus which permits user-defined types and polymorphic functions, and show that the semantics of this language satisfies a representation theorem which embodies our notion of a "correct" type structure.

Syntax

To formalize the syntax of our language, we begin with two disjoint, countably infinite sets: the set T of type variables and the set V of normal variables. Then W , the set of type expressions, is the minimal set satisfying:

(1a) If $t \in T$ then:

$t \in W$.

(1b) If $w_1, w_2 \in W$ then:

$(w_1 \rightarrow w_2) \in W$.

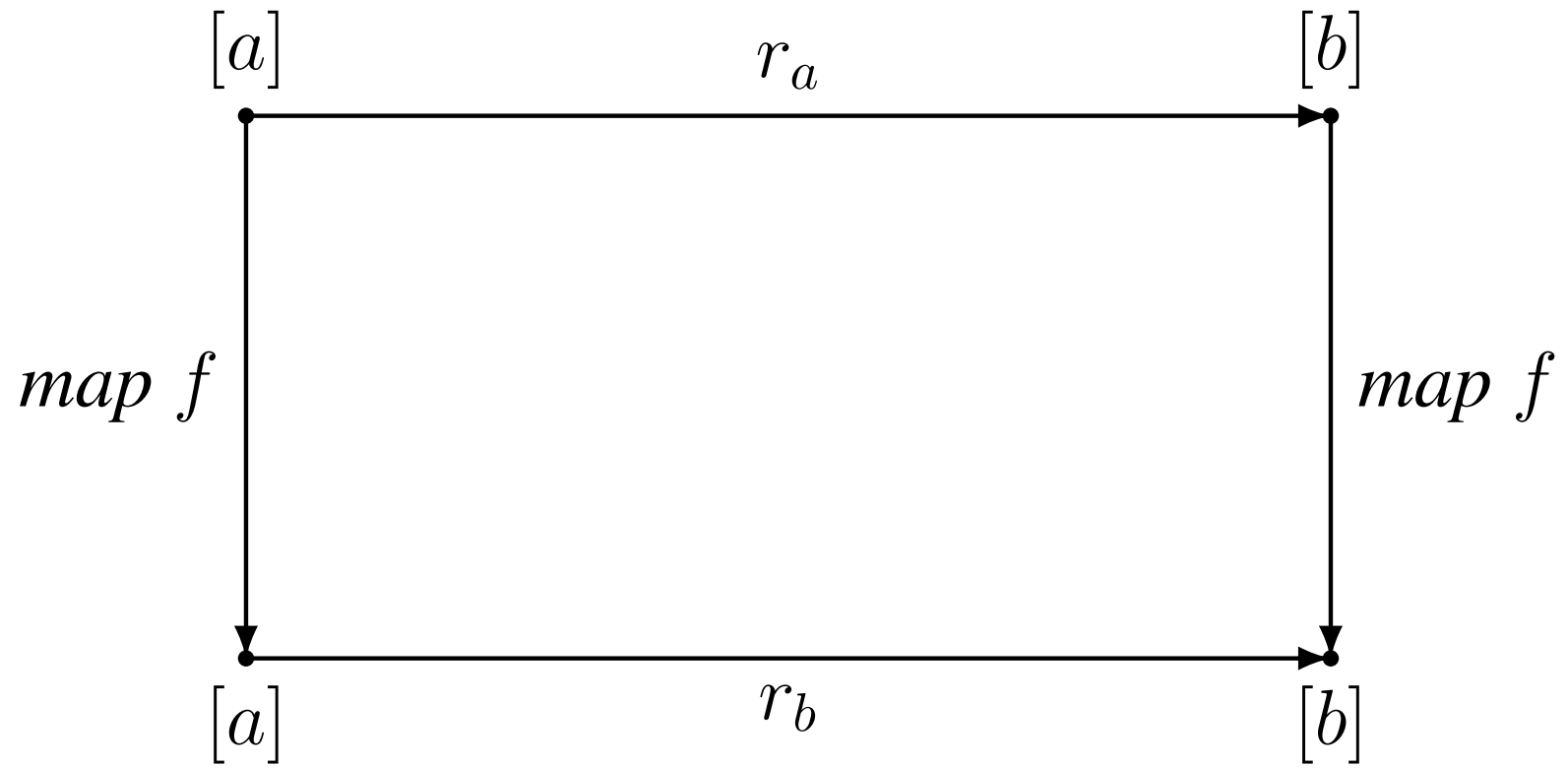
(1c) If $t \in T$ and $w \in W$ then:

$(\Delta t. w) \in W$.

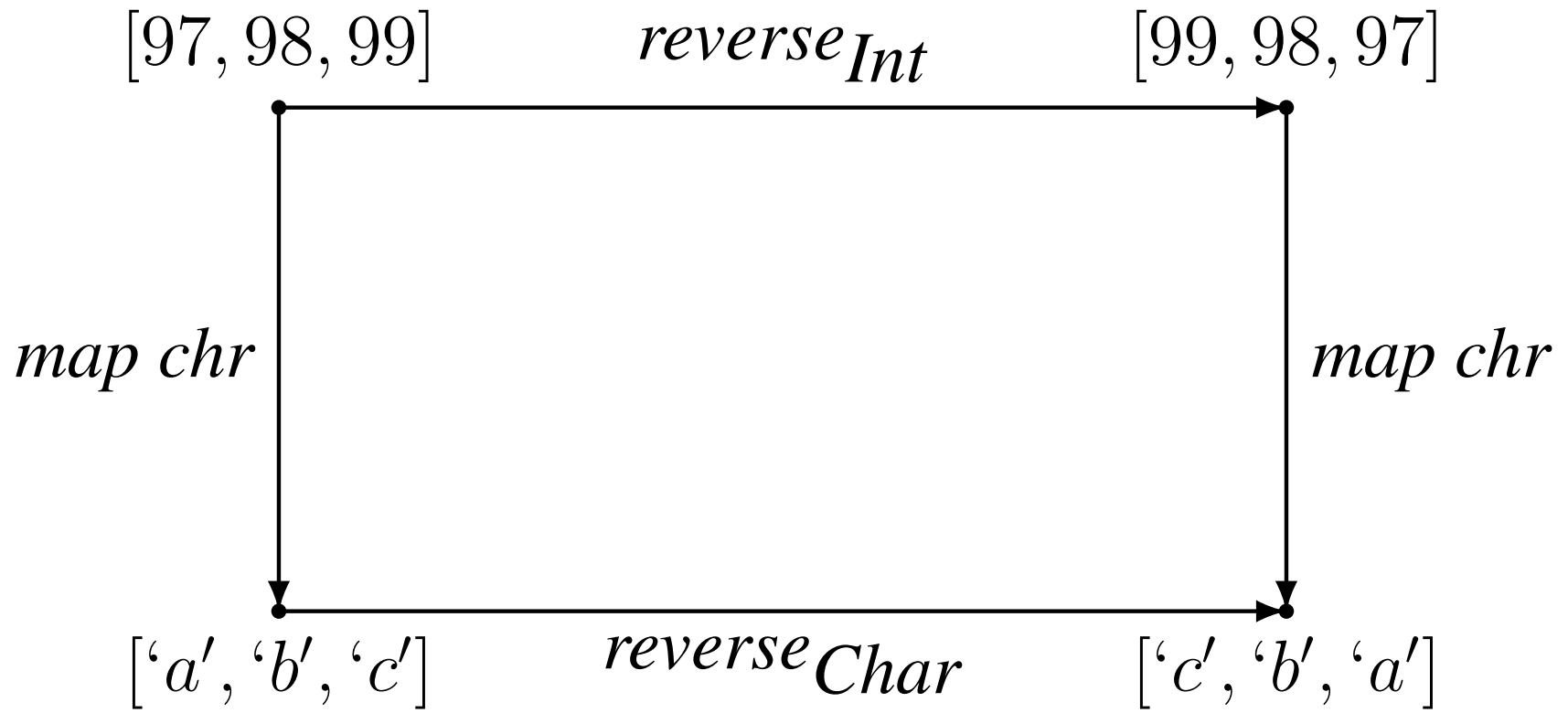
A magic trick

$$r :: [a] \longrightarrow [a]$$

Theorems for Free!



Theorems for Free!



Odersky and Wadler (1997) — Pizza

Pizza into Java: Translating theory into practice

Martin Odersky
University of Karlsruhe

Philip Wadler
University of Glasgow

Example 2.1 Polymorphism in Pizza

```
class Pair<elem> {
    elem x; elem y;
    Pair (elem x, elem y) {this.x = x; this.y = y;}
    void swap () {elem t = x; x = y; y = t;}
}

Pair<String> p = new Pair("world!", "Hello,");
p.swap();
System.out.println(p.x + p.y);

Pair<int> q = new Pair(22, 64);
q.swap();
System.out.println(q.x - q.y);
```

Example 2.3 Homogenous translation of polymorphism into Java

```
class Pair {
    Object x; Object y;
    Pair (Object x, Object y) {this.x = x; this.y = y;}
    void swap () {Object t = x; x = y; y = t;}
}

class Integer {
    int i;
    Integer (int i) { this.i = i; }
    int intValue() { return i; }
}

Pair p = new Pair((Object)"world!", (Object)"Hello,");
p.swap();
System.out.println((String)p.x + (String)p.y);

Pair q = new Pair((Object)new Integer(22),
                  (Object)new Integer(64));
q.swap();
System.out.println(((Integer)(q.x)).intValue() -
                  ((Integer)(q.y)).intValue());
```

Igarashi, Pierce, and Wadler (1999)

— Featherweight Java

$$\Gamma \vdash x : \Gamma(x)$$

$$\frac{\Gamma \vdash e_0 : C_0 \quad \text{fields}(C_0) = \bar{C} \ \bar{f}}{\Gamma \vdash e_0.f_i : C_i}$$

$$\frac{\Gamma \vdash e_0 : C_0 \quad \text{mtype}(\mathbf{m}, C_0) = \bar{D} \rightarrow C \quad \Gamma \vdash \bar{e} : \bar{C} \quad \bar{C} \triangleleft \bar{D}}{\Gamma \vdash e_0.\mathbf{m}(\bar{e}) : C}$$

$$\frac{\text{fields}(C) = \bar{D} \ \bar{f} \quad \Gamma \vdash \bar{e} : \bar{C} \quad \bar{C} \triangleleft \bar{D}}{\Gamma \vdash \mathbf{new} \ C(\bar{e}) : C}$$

$$\frac{\Gamma \vdash e_0 : D \quad D \triangleleft C}{\Gamma \vdash (C)e_0 : C}$$

$$\frac{\Gamma \vdash e_0 : D \quad C \triangleleft D \quad C \neq D}{\Gamma \vdash (C)e_0 : C}$$

$$\frac{\Gamma \vdash e_0 : D \quad C \not\triangleleft D \quad D \not\triangleleft C \quad \text{stupid warning}}{\Gamma \vdash (C)e_0 : C}$$

Igarashi, Pierce, and Wadler (1999)

— Featherweight Generic Java

$$\Delta; \Gamma \vdash x : \Gamma(x)$$

$$\frac{\Delta; \Gamma \vdash e_0 : T_0 \quad \text{fields}(\text{bound}_\Delta(T_0)) = \bar{T} \ \bar{f}}{\Delta; \Gamma \vdash e_0.f_i : T_i}$$

$$\frac{\begin{array}{c} \Delta; \Gamma \vdash e_0 : T_0 \quad \text{mtype}(\mathfrak{m}, \text{bound}_\Delta(T_0)) = \langle \bar{Y} \triangleleft \bar{P} \rangle \bar{U} \rightarrow U \\ \Delta \vdash \bar{V} \text{ ok} \quad \Delta \vdash \bar{V} \triangleleft [\bar{V}/\bar{Y}] \bar{P} \quad \Delta; \Gamma \vdash \bar{e} : \bar{S} \quad \Delta \vdash \bar{S} \triangleleft [\bar{V}/\bar{Y}] \bar{U} \end{array}}{\Delta; \Gamma \vdash e_0.\mathfrak{m} \langle \bar{V} \rangle (\bar{e}) : [\bar{V}/\bar{Y}] U}$$

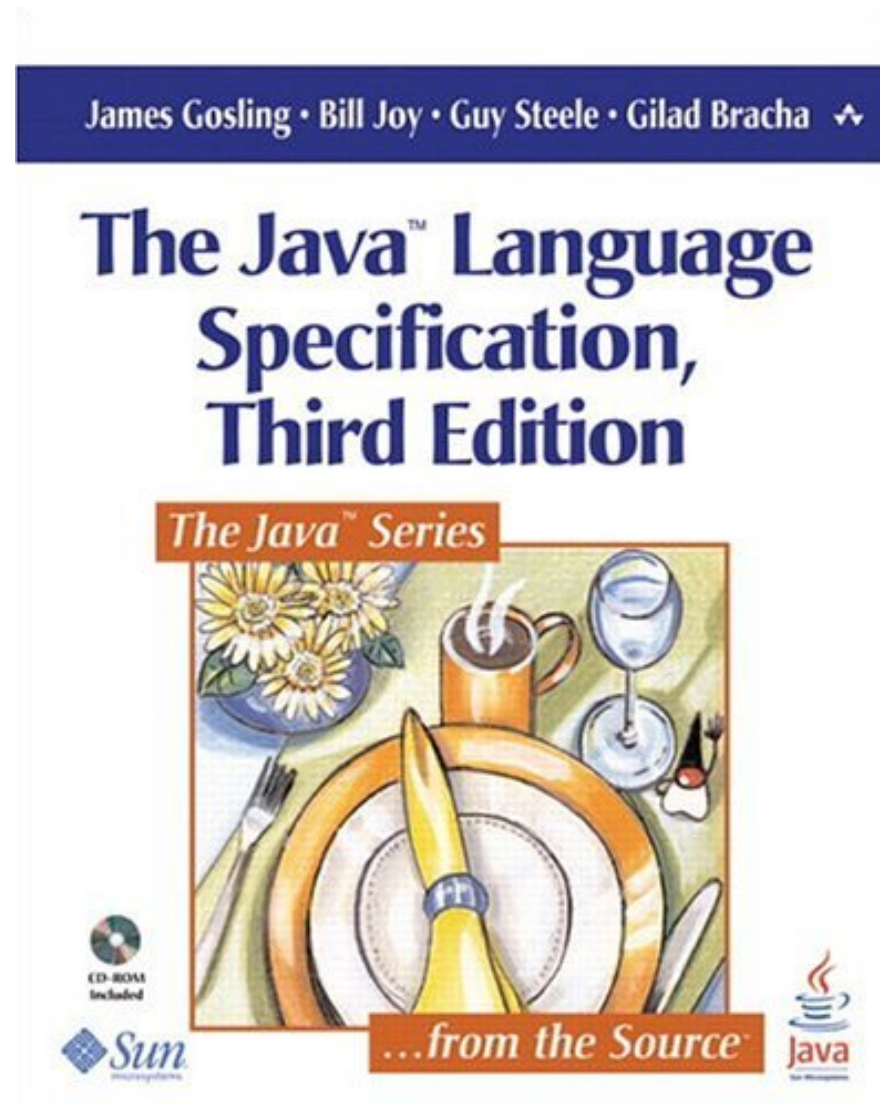
$$\frac{\Delta \vdash N \text{ ok} \quad \text{fields}(N) = \bar{T} \ \bar{f} \quad \Delta; \Gamma \vdash \bar{e} : \bar{S} \quad \Delta \vdash \bar{S} \triangleleft \bar{T}}{\Delta; \Gamma \vdash \text{new } N(\bar{e}) : N}$$

$$\frac{\Delta; \Gamma \vdash e_0 : T_0 \quad \Delta \vdash \text{bound}_\Delta(T_0) \triangleleft N}{\Delta; \Gamma \vdash (N)e_0 : N}$$

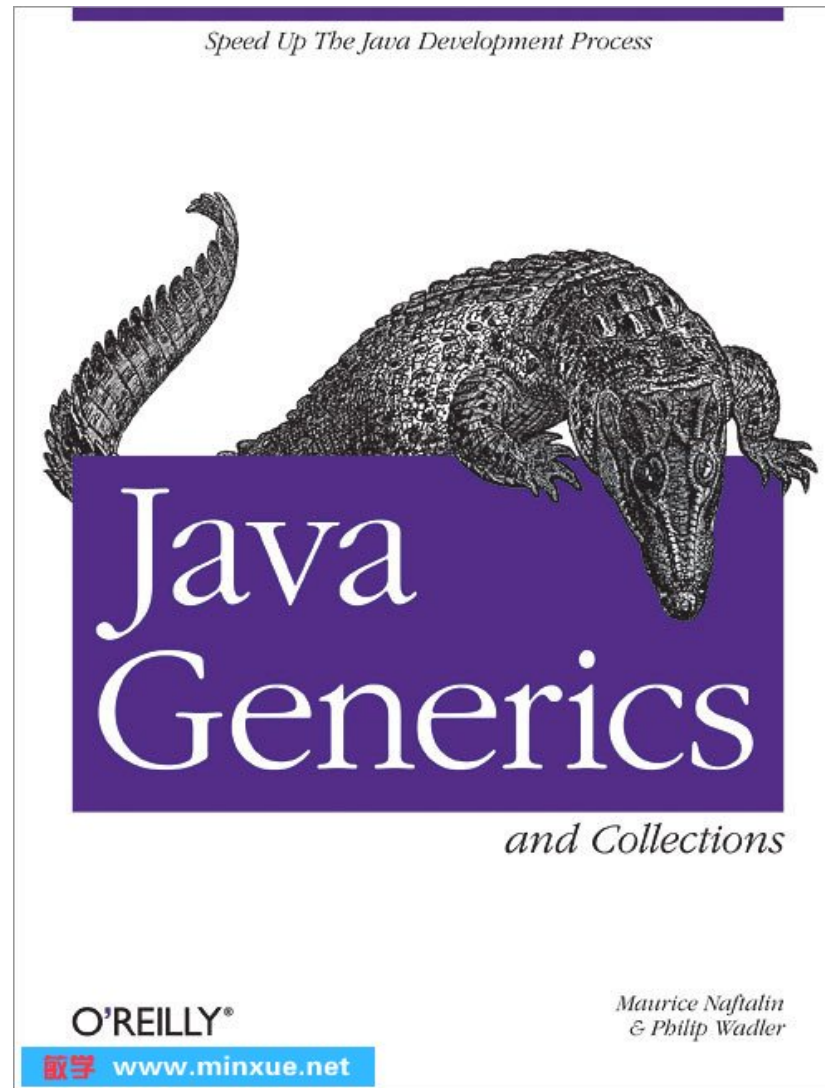
$$\frac{\begin{array}{c} \Delta; \Gamma \vdash e_0 : T_0 \quad \Delta \vdash N \text{ ok} \quad \Delta \vdash N \triangleleft \text{bound}_\Delta(T_0) \\ N = C \langle \bar{T} \rangle \quad \text{bound}_\Delta(T_0) = D \langle \bar{U} \rangle \quad \text{dcast}(C, D) \end{array}}{\Delta; \Gamma \vdash (N)e_0 : N}$$

$$\frac{\begin{array}{c} \Delta; \Gamma \vdash e_0 : T_0 \quad \Delta \vdash N \text{ ok} \quad N = C \langle \bar{T} \rangle \quad \text{bound}_\Delta(T_0) = D \langle \bar{U} \rangle \\ C \not\leq D \quad D \not\leq C \quad \text{stupid warning} \end{array}}{\Delta; \Gamma \vdash (N)e_0 : N}$$

Gosling, Joy, Steele, Bracha (2004) — Java 5



Naftalin and Wadler (2006)



Part III

Three recent ideas

Idea I: Object to Object

- Object vitiates parametricity

```
class Object {  
    Bool eq(Object that) {...}  
    String show() {...}  
}
```

- Top preserves parametricity

```
class Top {  
    // no methods!  
}
```

Idea II: Blame calculus

$$v : A \rightarrow B \Rightarrow^p A' \rightarrow B' \longrightarrow \lambda x' : A'. (v (x' : A' \Rightarrow^{\bar{p}} A) : B \Rightarrow^p B')$$

$$v : G \Rightarrow^p G \longrightarrow v$$

if $G \neq \star \rightarrow \star$

$$v : A \Rightarrow^p \star \longrightarrow v : A \Rightarrow^p G \Rightarrow \star$$

if $A \prec G$ and $A \neq \star$

$$v : G \Rightarrow \star \Rightarrow^p A \longrightarrow v : G \Rightarrow^p A$$

if $G \prec A$

$$v : G \Rightarrow \star \Rightarrow^p A \longrightarrow \text{blame } p$$

if $G \not\prec A$

$$(v : G \Rightarrow \star) \text{ is } G \longrightarrow \text{true}$$

$$(v : H \Rightarrow \star) \text{ is } G \longrightarrow \text{false}$$

if $G \neq H$

$$E[\text{blame } p] \longmapsto \text{blame } p$$

if $E \neq [\cdot]$

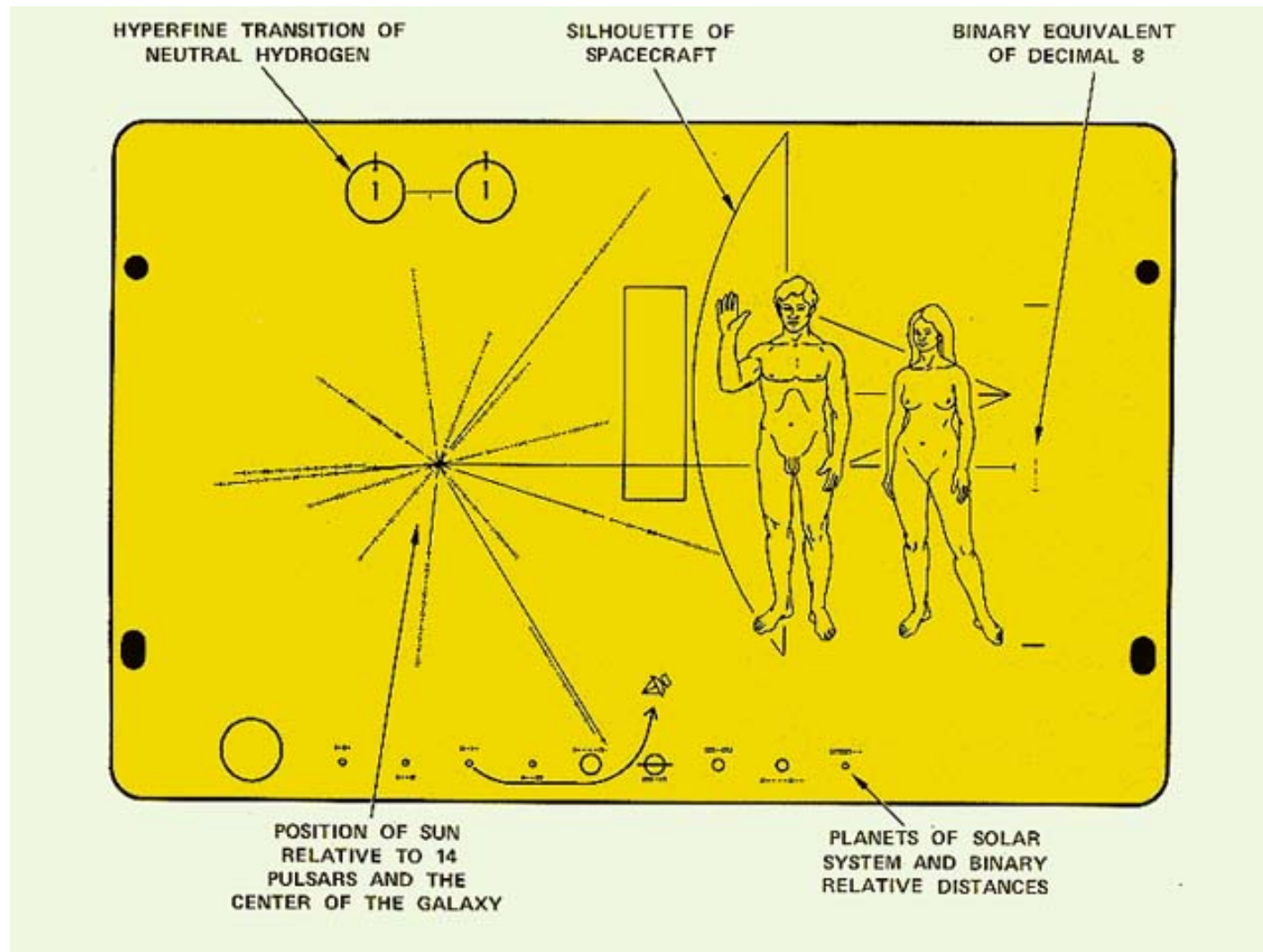
Idea III: Propositions as Sessions

$$\begin{array}{c}
 \frac{}{w \leftrightarrow x \vdash w : A^\perp, x : A} \text{Ax} \quad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^\perp}{\nu x : A. (P \mid Q) \vdash \Gamma, \Delta} \text{Cut} \\
 \\
 \frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B} \otimes \quad \frac{R \vdash \Theta, y : A, x : B}{x(y).R \vdash \Theta, x : A \wp B} \wp \\
 \\
 \frac{P \vdash \Gamma, x : A}{x[\text{inl}].P \vdash \Gamma, x : A \oplus B} \oplus_1 \quad \frac{P \vdash \Gamma, x : B}{x[\text{inr}].P \vdash \Gamma, x : A \oplus B} \oplus_2 \quad \frac{Q \vdash \Delta, x : A \quad R \vdash \Delta, x : B}{x.\text{case}(Q, R) \vdash \Delta, x : A \& B} \& \\
 \\
 \frac{P \vdash ?\Gamma, y : A}{!x(y).P \vdash ?\Gamma, x : !A} ! \quad \frac{Q \vdash \Delta, y : A}{?x[y].Q \vdash \Delta, x : ?A} ? \\
 \\
 \frac{Q \vdash \Delta}{Q \vdash \Delta, x : ?A} \text{Weaken} \quad \frac{Q \vdash \Delta, x : ?A, x' : ?A}{Q\{x/x'\} \vdash \Delta, x : ?A} \text{Contract}
 \end{array}$$

Part IV

Aliens

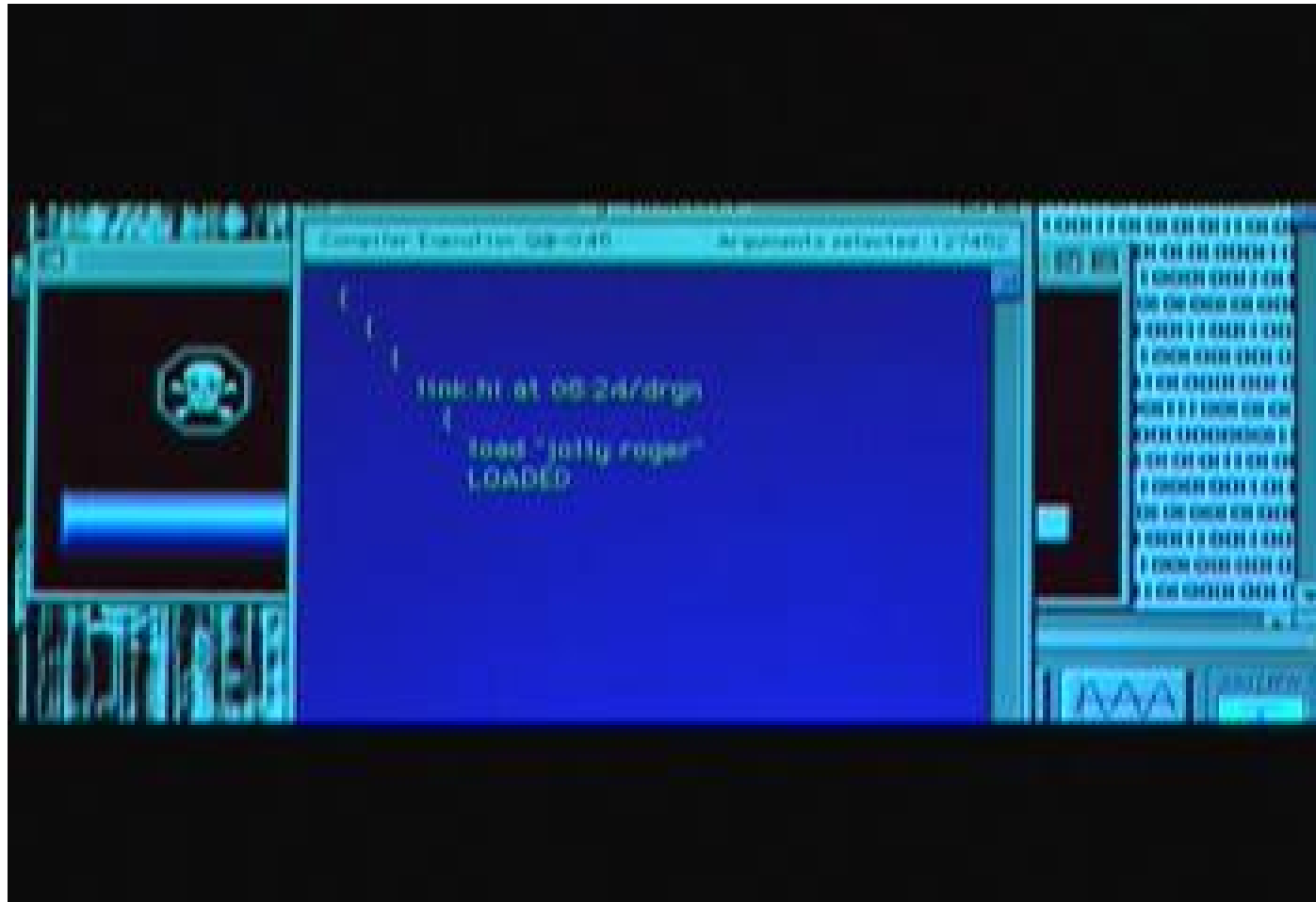
How to talk to aliens



Independence Day



A universal programming language?



Lambda is Omniversal

